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Start-up and steady thermal oscillation of a pulsating heat pipe

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Abstract As a novel electronic cooling device, pulsating heat pipes (PHPs) have been received attention in recent years. However, literature survey shows that no studies were carried out on the start-up and steady thermal oscillation of the PHPs. In the present paper, the copper capillary tube was being bended to form the snake-shaped PHP. Heating power was applied on the heating section, and transferred to the condensation section and dissipated to the environment by the pure natural convection. The inside diameter of the capillary tube is 2.0 mm and the working fluid is selected as FC-72. A high speed data acquisition system was used to detect the start-up and steady thermal oscillation of the PHP. Two types of the start-up process were observed: a sensible heat receiving start-up process accompanying an apparent temperature overshoot followed by the steady thermal oscillation at low heating power, and a smooth sensible heat receiving start-up process incorporating a smooth oscillation period at high heating power. For the steady thermal oscillation, also two types were found: the random thermal oscillation with a wide frequency range, indicating the random distribution of the vapor plug and liquid slug inside the capillary tube at low heating power, and the quasi periodic thermal oscillation with the same characteristic frequency for both heating section and condensation section, indicating the uniform distribution of the vapor plug and liquid slug inside the capillary tube at high heating power. The power spectral density (PSD) was used to analyze the thermal oscillation waves. The frequency corresponds to the time that a couple of adjacent vapor plug and liquid slug passing through a specific wall surface.

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List of symbols

d	inside tube diameter (m or mm)
g	gravity acceleration (m/s^2)
H_{fg}	latent heat of evaporation (J/kg)
P	pressure (Pa)
r	radius (m)
T	temperature (°C)
V	velocity of the vapor plug/liquid slug
ρ	density (kg/m^3)
σ	surface tension (N/m)
α	heat transfer coefficient (W/m^2 K)
ΔT	temperature overshoot

Subscripts

f	liquid state
liquid	liquid state
film	thin liquid film
g	vapor state
in	inner wall surface
sat	saturation
start-up	start-up process
w	wall

1 Introduction

In recent years, the fast development of the integrated circuit (IC), semiconductor industries make the necessary to develop novel electronic cooling devices. There are two types of the electronic cooling devices, the passive one and the active one. The passive electronic cooling devices, such as heat pipes and capillary pump loops, are widely applied due to their simple structures and high reliabilities. The concept of pulsating heat pipe (PHP) was first proposed by Akachi [1] in his patent. It is so named as PHP because it possesses all the advantages of the traditional heat pipes, such as simple configuration structure and low maintenance cost. On the other hand, the working principle of the PHP is quite different from the classical one. For a conventional heat pipe, the static phase change (boiling and condensation) heat transfer takes place inside. Boiling takes place in the heating section and condensation in the cooling section. The condensed liquid is flowing back to the boiling section through the wick structures. However, the distinct dynamic process of the PHPs makes the problem very complicated.

In the 10 years following 1990 when Akachi first proposed the PHPs, it seems that its research and development are very slow, we did not find any public literatures dealing with PHPs. The major information of the PHPs is coming from the recent International Heat Pipe Conferences. Lee et al. [2] performed the qualitative flow visualization using glass capillary tube PHP. Very recently Shafii et al. [3, 4] conducted the numerical modeling on PHP, and concluded that the sensible heat plays the major part of the total heat transfer. They also demonstrated that the gravity force has an insignificant effect on the PHP performance.

As to author's knowledge, there is no experimental/theoretical work on the start-up and steady thermal oscillation of the PHPs. In the present paper, we fabricated a PHP test section by bending the copper capillary tube turn by turn. The inside diameter of the capillary tube is 2.0 mm with the wall thickness of 0.5 mm. The working fluid is selected as FC-72. A high speed data acquisition system is used to detect the start-up and steady thermal oscillation of the PHPs. Two types of the start-up and steady thermal oscillations are identified. The power spectral density (PSD) method was used to analyze the periodic and random temperature waves. The present results are original and are helpful for the understanding of the PHPs.

2 PHP working principle

If the inside diameter of a capillary tube is small enough such that $d \leq 1.83\sqrt{\sigma/g(\rho_f - \rho_g)}$, vacuum the capillary tube then partially charge liquid can form the vapor plug and liquid slug distributed inside the capillary tube. Bending the capillary tube turn by turn can form the snake-shaped PHP. There are two types of PHP structures, the looped PHP and the unlooped PHPs. In our previous paper [5], we pointed out that the unlooped PHP, actually, is not helpful for the fluid circulation inside the PHP. Therefore we only presented the experimental results on the looped PHP. It is easy to understand that surface tension plays an important role on the performance of the PHP. Figure 1 is the PHP model that is used in the present study. Heating applied on the lower part of the parallel capillary tube causes the uneven pressure among the parallel tubes and induces the fluid movement in one direction. However, such fluid movement instantaneously changes the pressure distribution thus changes the fluid movement direction. Thus the oscillating flow is self-maintained. Heat is transferred from the heating section to the condensation section and dissipated to the environment by the pure natural convection through the upper part of the capillary tube. The present PHP is a purely passive cooling device.

3 PHP model and its test facility

The PHP model and its test facility, consisting of the PHP, the high speed data acquisition system, and the

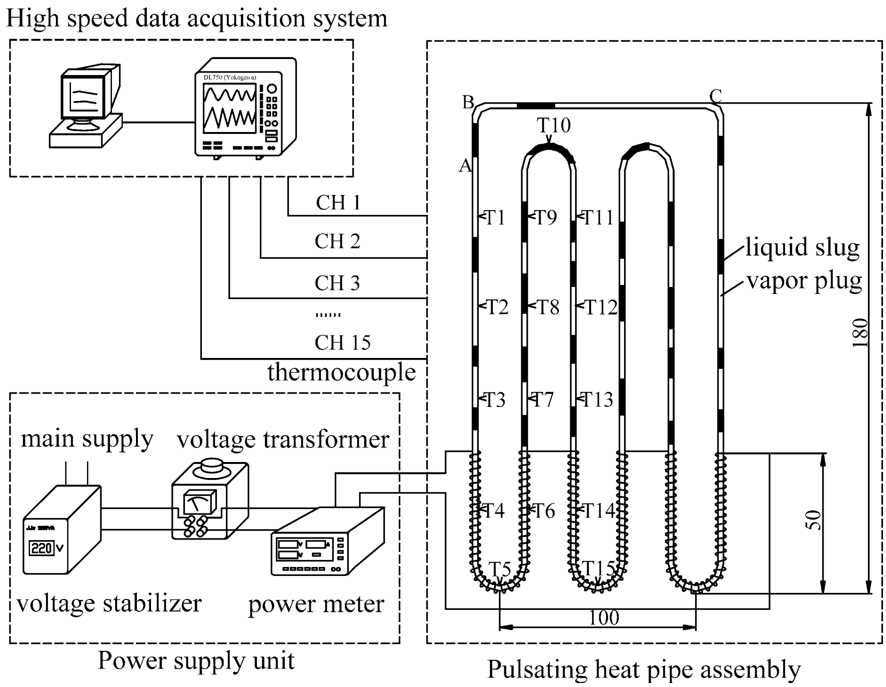


Fig. 1 PHP assembly and its experimental apparatus

Table 1 Thermocouple location along the stretched capillary tube (mm)

T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15
20	55	90	130	166	201	241	276	311	347	383	418	453	493	529

Table 2 The physical properties of FC-72 at 1 atm saturated condition

Working fluids	T_{sat} (°C)	ρ_f (kg/m^3)	ρ_g (kg/m^3)	$C_{p,f}$ (J/kg°C)	H_{fg} (kJ/kg)	σ (N/M)
FC-72	56.6	1,600	13.43	1,102	94.8	8.35×10^{-3}

power supply unit, are shown in Fig. 1. The PHP is made of copper capillary tube, with the total height of 180 mm and the width of 100 mm. The height of the heating section is 50 mm, with the other 130 mm directly exposed in the air environment above the heating section. The heating power is applied on the heating section by wrapping the Ni–Cr heating wire on the outer surface of the capillary tube with the axial interval of 1.5 mm. A total of 15 fine thermocouple wires with the diameter of 0.1 mm were welded on the outer surface of the PHP, covering the half of the whole PHP, due to the geometry symmetry. The locations of each thermocouple wires along the stretched capillary tube starting from point A are illustrated in Table 1. Above the heating section, the capillary tube is directly exposed in the air environment. Heat is received from the heating section and transferred to the upper section of the PHP and directly dissipated to the environment by the pure nature convection. The room temperature is kept as 23°C with 1°C uncertain by the air condition system.

The heating power is provided by a carefully designed power supply unit. The power meter measures the AC voltage, the current and the corresponding power simultaneously, with the uncertainty of 1%. A high speed data acquisition system with high resolution and accuracy (DL 750, YOKOGOWA Inc., Japan) is used to measure the dynamic thermal oscillation waves. The thermocouples are calibrated using the constant temperature bath incorporated with the data acquisition system. The accuracy of these thermocouples is within 0.1°C uncertainty. A vacuum/charging system, which is specifically designed and fabricated in our laboratory, was used to vacuum the PHP and charge a controlled amount of liquid into the PHP tube. The net liquid filled inside the PHP tube is decided by an electronic balance, with the uncertainty of 0.01 g. The fill ratio is defined as the net liquid volume divided by the total inside volume of the PHP. The fill ratio that is used in the present paper is 70%, PHPs with such fill ratio have the best thermal performance, based on our previous study [5]. The working fluid is selected as FC-72, which has small surface tension and low latent heat of evaporation. The physical properties at 1 atm saturated condition are listed in Table 2. Also based on our previous study [5], we found that PHP using FC-72 as working fluid has low activating power than that using DI water, which is useful for the PHP working at low heating power. This is the main reason why we choose FC-72 as the working fluid in the present paper.

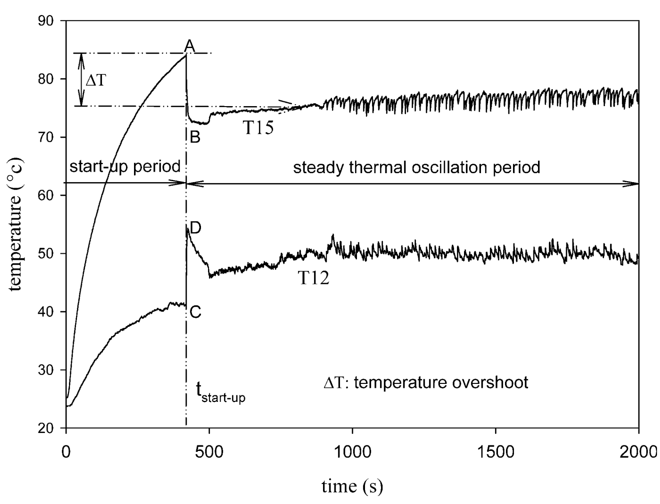


Fig. 2 Start-up of the PHP at low heating power of 10.0 W

4 Start-up process of the PHP

There is no experimental/theoretical work on the start-up of the PHPs available. In the present paper, we identified two types of the PHP start-up process.

4.1 Sensible heat receiving start-up process incorporating an apparent temperature overshoot

Initially the PHP is in the frozen state at the room temperature. Suddenly the heating power is applied on the heating section. We recorded the time series of the temperature oscillations from the frozen state. Figure 2 shows a typical time series of the PHP including the start-up process followed by the steady thermal oscillation, where T15 is located on the heating section and T12 on the condensation section. From $t=0$ to $t=t_{start\text{-}up}$ is the start-up process. The temperatures of T15 and T12 are increased versus time without any signal oscillations, indicating that the fluid movement is not initiated. The heating power is used for the sensible heat increment received by the copper capillary tube and the fluid inside. The curvature of T15 is caused by the axial heat conduction through the copper capillary tube. The temperature T12 is also increased in the start-up process, but the increment slope is much smaller that that of T15, also with apparent inverted curvature.

As shown in Fig. 2, from the start-up period to the steady thermal oscillation period is not a smooth transition. At $t=t_{start\text{-}up}$, the temperature T15 has a higher value of 85.0°C at point A, then sharply decreased to

75.0°C at point B, followed by the oscillation period. The temperature overshoot ΔT during such transition is 10.0°C. The temperature T15 has a sharp decrease from point A to B indicating that the fluid movement is initiated at $t=0$ to $t=t_{\text{start-up}}$. On the other hand, the initiation of the fluid movement at $t=0$ to $t=t_{\text{start-up}}$ leads to a sharp increase of the temperature T12 from point C to D, indicating the heat transfer from the heating section to the condensation section by the fluid motion.

The oscillating flow inside the PHP is coming from the instantaneous pressure differences among the parallel capillary tubes. The pressure in any capillary tube is balanced by the gravity force and the surface tension force. When the inside diameter of the capillary tube is small, the surface tension domains the flow and heat transfer. The pressure governed by the surface tension is related to the vapor plug distribution in the capillary tube. Initially the PHP is in frozen state, suddenly when the heating power is applied on the heating section, the heating section should receive enough energy that causes the deformed vapor plug shape in the capillary tube, leading to the pressure redistributed among the parallel capillary tubes. Once such energy accumulation process results in apparent pressure difference among the parallel capillary tubes, the fluid movement is initiated, thus heat can be transferred from the heating section to the cooling section. Following the time series after the PHP is activated, the PHP can sustain the self-oscillating flow. The wall temperatures in the heating section are oscillating against mean values which are lower than the maximum values, that are obtained during the start-up process. This is the explanation of the temperature overshot phenomenon.

The temperature overshoot phenomenon is important for the PHP real applications. For the IC chip, under most circumstances, the temperature should be limited less than 84°C. Higher temperature larger than that value may cause the overheating of the IC chip, resulting in the bad performance of the IC chip, even leading to the burn-out of the chip. Therefore such temperature overshoot should be avoided during the start-up process of the PHP application.

4.2 Smooth start-up process of the PHP at higher heating power

The start-up process of the PHP at higher heating power is quite different from that at lower heating power. Such typical time series of the temperatures are illustrated in Fig. 3 for the heating power of 25.0 W. Generally three periods are divided. The first one is the start-up period for $t \leq t_A$, which is similar to those at lower heating power, but without the temperature overshoot. Again the fluid inside PHP is stationary without oscillating movement. The second period starts from $t=t_A$ and ends at $t=t_B$, during which the fluid flow inside is initiated and the temperature oscillation is detected. However, the temperatures in the heating section (T15) and condensation section (T12) are still increased versus time, but with much slower increment slope than those in the first period for $t \leq t_A$. The third period starts from $t=t_B$, the PHP begins to evolve into the steady thermal oscillation. Generally the start-up process incorporates the first and second periods, from $t=0$ across $t=t_A$ to $t=t_B$.

5 PHP thermal oscillation

5.1 Analysis of the PHP thermal oscillation

Phase change heat transfer is widely applied in the energy and chemical engineering. Depending on the operation condition, two-phase system may display the pressure drop oscillation, the density wave oscillation and the thermal oscillation. Thermal oscillation is caused by the flow rate oscillation thus the heat transfer coefficient between the fluid and the wall surface may vary periodically. However, literature survey shows that very little studies were carried out on the thermal oscillation when the channel size is down to 2.0 mm. In this paper, we analyze the reason of the thermal oscillation occurring in the PHP. Through the simple computation we understand that the temperature difference between the outer and inner surface is very small due to the large thermal conductivity of the copper capillary tube and the thin wall surface. Such temperature difference is less than 0.1°C for all the run cases encountered in the present paper.

5.1.1 In the heating section, the local wall surface is temporarily immersed in the vapor plug region, as shown in Fig. 4a

Figure 4 shows the vapor plug and liquid slug coexisting in the capillary tube. The local wall surface is either exposed in the vapor plug region, or the liquid

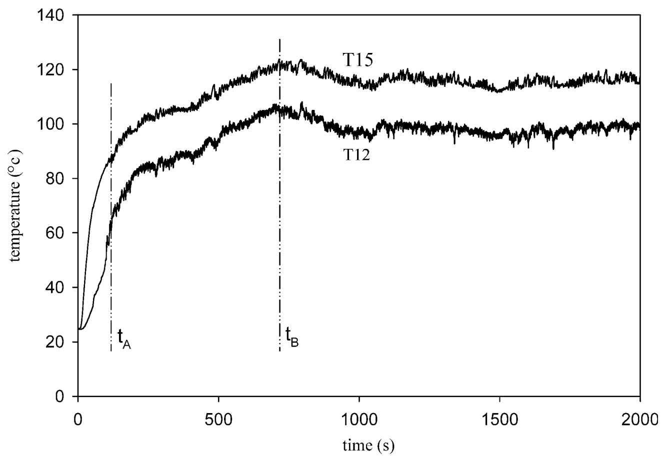


Fig. 3 Start-up of the PHP at high heating power of 25.0 W (first period for $0<t<t_A$, second period for $t_A<t<t_B$, third period for $t>t_B$)

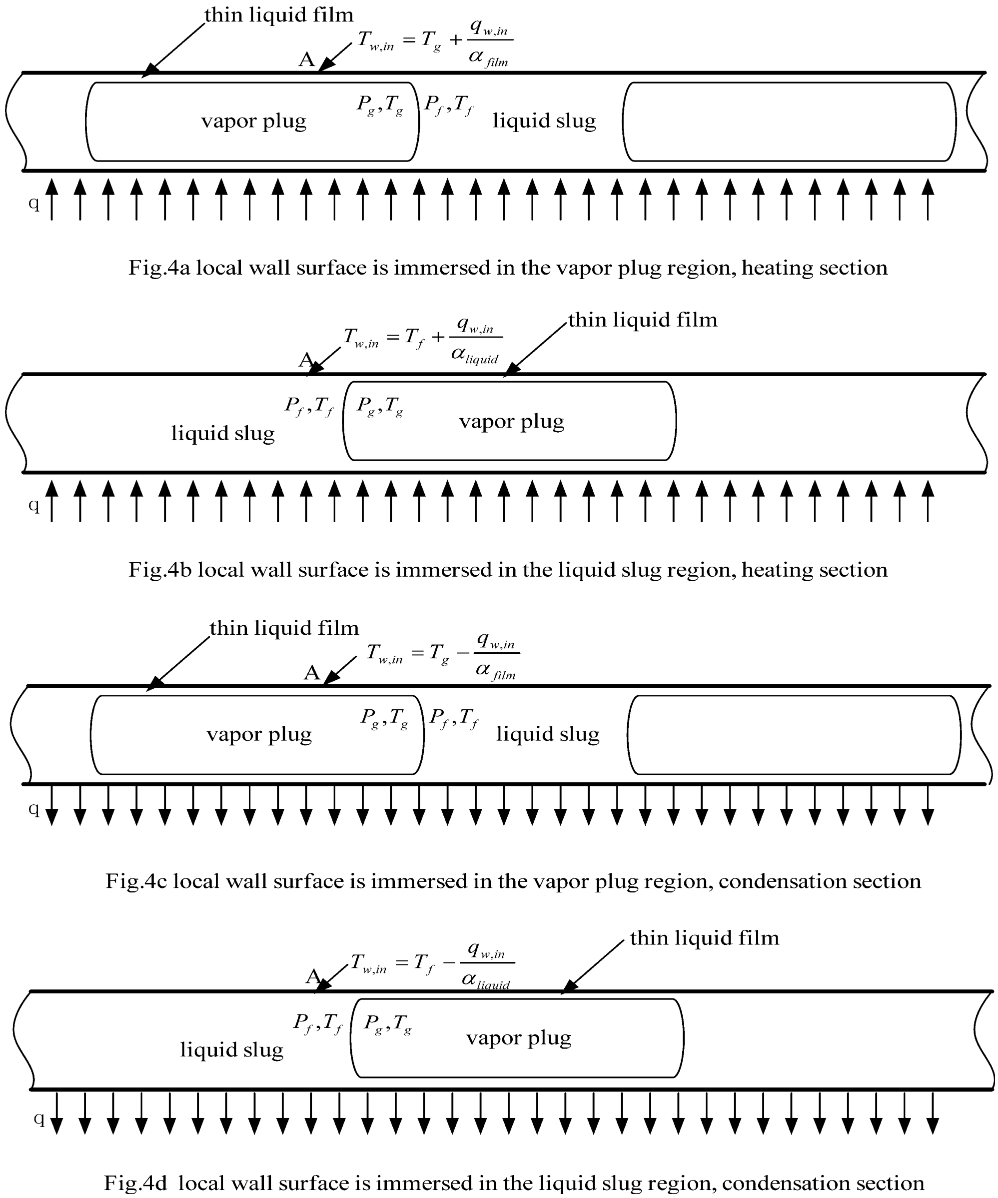


Fig. 4 **a** Local wall surface is immersed in the vapor plug region, heating section. **b** Liquid slug region, heating section. **c** Vapor slug region, condensation section. **d** Liquid slug region, condensation section

slug region. In the vapor plug region, a thin liquid film separates the vapor plug body and the tube wall surface, providing a higher boiling heat transfer coefficient. Across the vapor/liquid interface where the evaporation heat transfer takes place, the pressure difference across the vapor side and the liquid side is written as

$$P_g - P_f = \frac{2\sigma}{r} \tag{1}$$

where r is the meniscus radius of the vapor/liquid interface. The vapor plug temperature T_g should be larger than the saturated temperature corresponding to the pressure P_g. That is $T_g \geq T_{sat}$ (P_g). Because heat is transferred from the liquid to the vapor, the liquid temperature T_f should be larger than T_g, that is

$$T_f > T_g \geq T_{sat}(P_g) \tag{2}$$

The local wall surface temperature is

$$T_{w,in} = T_g + \frac{q_{w,in}}{\alpha_{film}} \tag{3}$$

where α_{film} is the heat transfer coefficient due to the thin film evaporation.

5.1.2 In the heating section, the local wall surface is temporarily immersed in the liquid slug region, as shown in Fig. 4b

Under such condition, the local wall surface temperature is

$$T_{w,in} = T_f + \frac{q_{w,in}}{\alpha_{liquid}} \tag{4}$$

where α_{liquid} is the liquid forced convection heat transfer coefficient, which is smaller than α_{film}. Because $T_f > T_g$ and $\alpha_{liquid} < \alpha_{film}$, the local wall surface temperature when it is immersed in the liquid slug region in terms of Eq. 4 should be larger than that when it is immersed in the vapor plug region by Eq. 3.

For an operating PHP, any local wall surface is flushed by the vapor plug and the liquid slug alternatively. The local wall temperature displays higher value when it is flushed by the liquid slug, while it behaves lower value when it is flushed by the vapor plug. Such process repeats periodically, forming the thermal oscillation of the wall surface.

In the condensation section, an inverse process takes place, as shown in Fig.4c, d. Because the condensation heat transfer occurs at the vapor/liquid interface, the vapor plug should have higher temperatures than the liquid slug. Therefore the wall surface has higher temperatures when it is immersed in the vapor plug region, while it has lower values when it is flushed by the liquid slug. The thermal oscillation is caused by the local surface alternatively immersed in the vapor plug and the liquid slug region. The process repeats and the thermal oscillation occurs.

5.1.3 *Power spectral density analysis of the steady thermal oscillation of the PHP*

As described in the above section, in the heating section, the temperature is increased when the local wall surface is flushed by the liquid slug, while it is decreased when the same place is flushed by the vapor plug. Considering an assumed temperature signal shown in Fig. 5, a typical full cycle can be considered as from t_A to t_C, or from t_B to t_D. Again assuming the instantaneous flow direction is from right to left in the heating section, t_A corresponds to the time when the local wall surface begins to be contacted with the front of the liquid slug 1, thus the local temperature begins to be increased, as analyzed in the above section. After a certain time lapsed, the local wall surface is on the tail of the liquid slug 1 at t_B. Following t_B, the local wall surface is contacted with the front of the vapor plug 2, thus the temperature is decreased, until the local wall surface is on the tail of the vapor plug 2. It is seen that a full cycle should contain the neighboring vapor plug and liquid slug passing through a specific local wall surface. Therefore the cycle time period (or frequency) of the temperature signal corresponds to the time that a couple of adjacent vapor plug and liquid slug passing through the specific wall surface.

$$t_{cycle} = \frac{L_{vapor\ plug} + L_{liquid\ slug}}{V} \tag{5}$$

where V is the velocity of the vapor plug or the liquid slug passing through the local wall surface.

Power spectral density is usually applied to extract the periodic feature of a signal. Wang et al. [6] computed the PSD of the differential pressure fluctuations to distinguish the two-phase flow through the T-junctions between periodic and random motion. If the power spectrum is continuous and asymptotic, the motion can be considered as random [7]. However, if the power spectrum only has a sharp peak distribution at a dominant characteristic frequency, the motion can be considered as periodic. Considering the power spectrum distribution through the Fourier transform, the higher power spectrum distribution corresponds to that there is a larger possibility at which the motion is in its corresponding frequencies.

5.2 Thermal oscillation measurements and their PSD analysis

5.2.1 *Low heating power conditions*

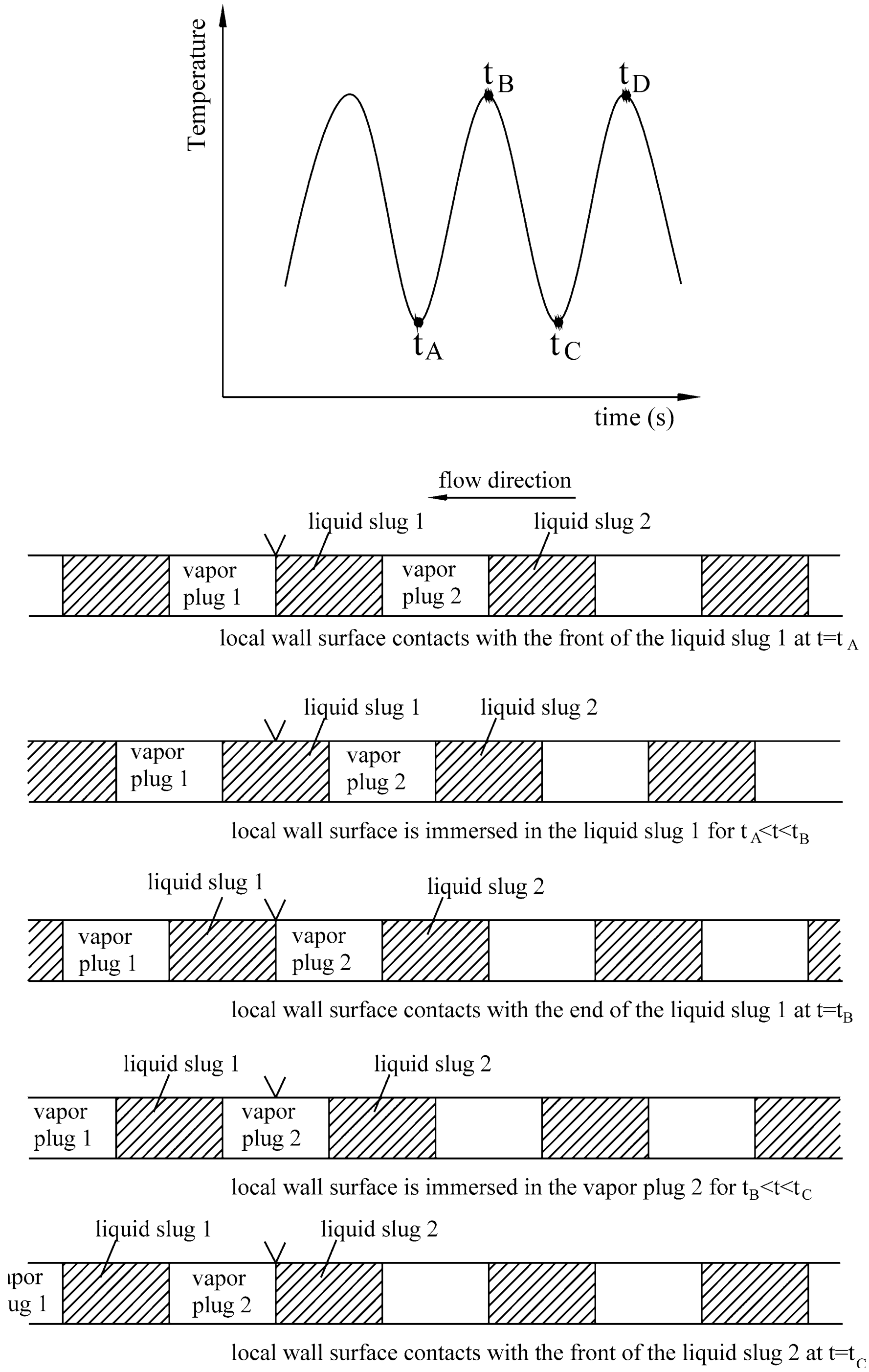
Figure 6 shows the steady thermal oscillation of the PHP at the heating power of 12.0 W, for the temperatures T5 (heating section), T7 and T9 (condensation section). It is seen that the temperature signals of T7 and T9 are more random than that of T5. The mean oscillation amplitudes of T5, defined as the peak value against the average mean value for each cycle, is from 1.5 to 2.0°C for most cycles. While such oscillation amplitudes of T7 and T9 (condensation section) are less than 1.0°C, which are smaller than those of T5. The higher temperature oscillation amplitudes in the heating section are caused by the higher heating flux in the heating section than those in the condensation section. The power spectrum distribution versus the frequencies is illustrated in Fig. 7 for the temperature T5. It is seen that such power spectrum distribution has a frequency range from 0 to 0.35 Hz, but with a dominant frequency of 0.093 Hz, corresponding to the dominant time period of 10.76 s. It is interesting to note that the dominant time period of 10.76 s from the PSD analysis is consistent with the average time period computed from the total time of 140.0 s covered by 13 cycles (see Fig. 6 for T5 oscillation).

As analyzed above, the characteristic frequency corresponds to the characteristic time that a couple of adjacent vapor plug and liquid slug passing through a specific local wall surface. The lower frequencies (or the longer time periods) may be explained as that the vapor plug and the liquid slug distributed in the capillary tube are long at the lower heating power. Also, the frequency distribution over a wider range indicates that the motion is random, and the vapor plug and liquid slug are nonuniformly distributed in the capillary tube at the lower heating power.

5.2.2 *High heating power condition*

However, the steady thermal oscillations at high heating power are quite different from those at the lower heating power. A typical thermal oscillation of T5, T7

Fig. 5 Schematic illustration of the temperature response for a full cycle



and T9 at the heating power of 25.6 W is shown in Fig. 8. From the appearance it is seen that such oscillations are quasi periodic. The PSD analysis of T5, T7 and T9 are shown in Fig. 9. Distinct with those at the lower heating power of 12.0 W, the PSD analyses of T5, T7 and T9 have three, two and one characteristic frequencies, respectively. The power spectrums for these signals are not continuous, but have peak values at these characteristic frequencies, therefore it is reasonable to consider the motion inside the capillary tube as quasi periodic. The first dominant frequency for T5 is very low, further, such first dominant frequency for T7 is close to zero. Such very low dominant frequencies may correspond to the very slow transition of the temperature oscillation, due to the slow changes of the environment conditions, such as the small deviation of the heating power, or the small change of the air environment temperature which affects the pure natural convection heat transfer on the upper section of the PHP. From Fig. 9, it is interesting to note that the second characteristic frequencies are 0.46 Hz, both for T5 and T7, which are also equal to the single

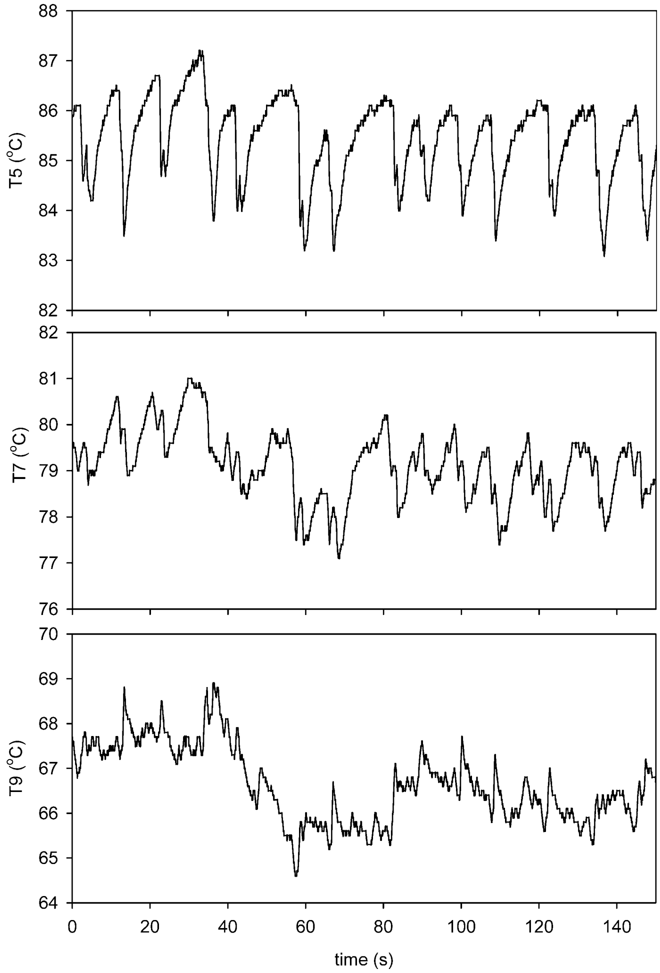


Fig. 6 Steady thermal oscillation of the PHP at the heating power of 12.0 W

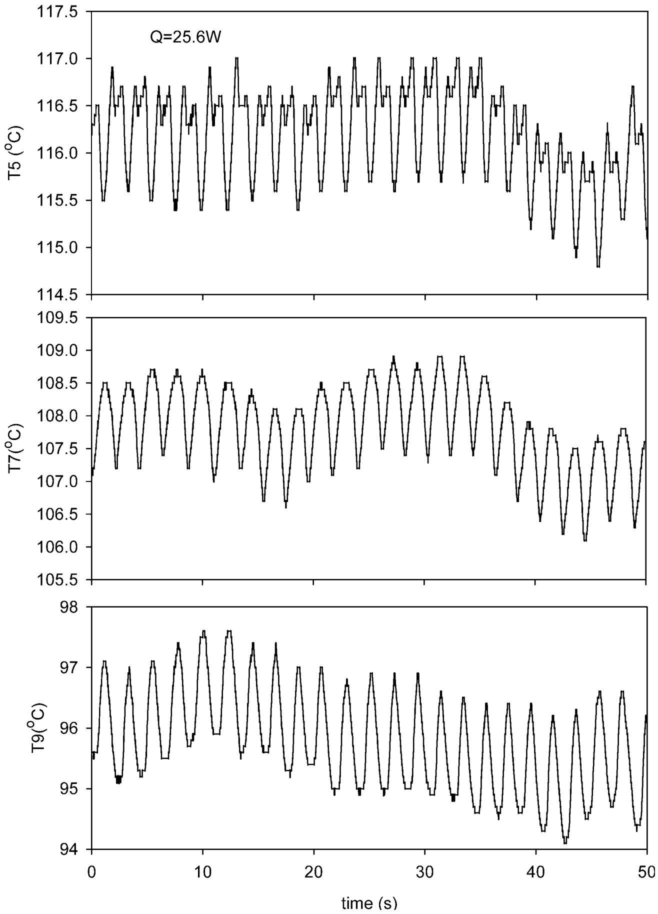


Fig. 8 Steady thermal oscillation of the PHP at the heating power of 25.6 W

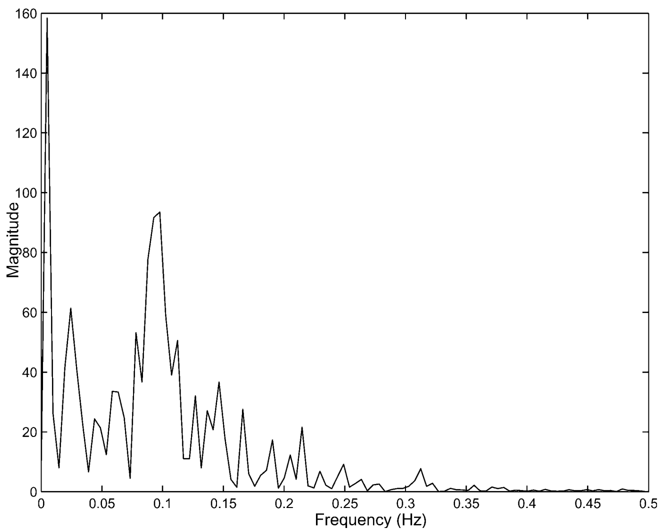


Fig. 7 Power spectrum analysis of T5 of the PHP at the heating power of 12.0 W

characteristic frequency for T9. The power spectrums have dominant higher magnitude values at the dominant frequency of 0.46 Hz. The third dominant frequency of 0.9 Hz, nearly two times of the second dominant frequency of 0.46 Hz for T5, should correspond to the shorter time period oscillation imposed on the signal oscillations at the second dominant frequency.

The major dominant characteristic frequency of 0.46 Hz for all the three temperature signals gives the evidence that the fluid in the capillary tube is oscillating at the same characteristic frequency. The oscillating flow can be considered as quasi periodic, inferring that the vapor plug and liquid slug are quasi uniformly distributed in the capillary tube at the high heating power. In other words, higher heating power makes the distribution of the vapor plug and the liquid slug more uniform.

For a closed loop PHP, the total liquid amount in the snaked-shape capillary tube is constant for all the working conditions, even though the local distribution of the vapor plug and liquid slug (two-phase structure) is quite different for the low heating power and high

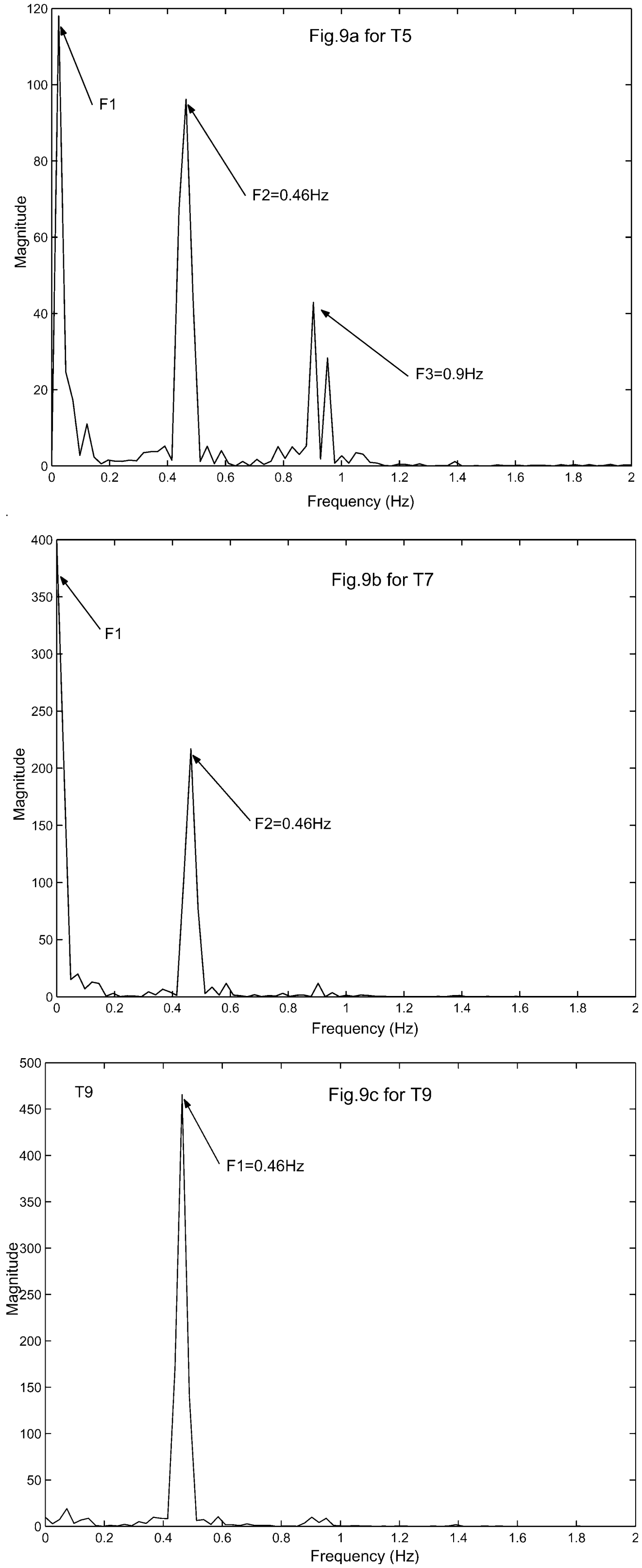


Fig. 9 Power spectrum analysis of the PHP at the heating power of 25.6 W

heating power. Based on our recent high speed flow visualization of the PHPs, the flow pattern in the capillary tube is the dispersed bubble, bubble slug and long bubble slug. The bubble slug/long bubble slug has a thin liquid film separating the bubble slug body and the wall surface, which has inverse liquid film velocity of the moving bubble slug. Considering the liquid fill ratio of 0.7, the corresponding mean volume void fraction is 0.3. Under such low volume void fraction, annular flow never happens for all the heating powers. The present study using the PSD analysis over the dynamic temperature signal illustrates that the non-uniform distribution of the vapor plug at the low heating power and the quasi uniform distribution of the vapor plug at high heating power.

6 Conclusions

An experiment was performed with a PHP, which was made of the copper capillary tube with the inside diameter of 2.0 mm. Heating power was applied on the heating section, and transferred to the condensation section and dissipated to the environment by the pure natural convection. The start-up process and the steady thermal oscillations were recorded by a high speed data acquisition system. The following conclusions can be drawn:

1. Two types of the start-up process were observed: the sensible heat receiving start-up process with fluid stationary inside accompanying an apparent temperature overshoot at low heating power, the sensible heat receiving process without fluid motion inside incorporating a smooth oscillation transition period with oscillation flow at high heating power.
2. The power spectrum analysis was performed to analyze the thermal oscillation of the PHP. The frequency corresponds to the time that a couple of adjacent vapor plug and liquid slug passing through a specific wall surface.
3. The oscillation flow at low heating power displays the random behavior, with a wider frequency range, but still with a dominant frequency, indicating the random distribution of the vapor plug and liquid slug inside the capillary tube.
4. The oscillation flow at high heating power displays the quasi periodic behavior. The power spectrum is not continuous, but has a couple of dominant frequencies. It is interesting to note that for the temperatures in the heating section and condensation section can share a same dominant characteristic frequency, indicating the quasi uniform distribution of the vapor plug and liquid slug inside the capillary tube.

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