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Effects of inner iteration times on the performance of IDEAL algorithm ☆

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ABSTRACT

An efficient segregated algorithm for non-linear incompressible fluid flow and heat transfer problems, called IDEAL (Inner Doubly-Iterative Efficient Algorithm for Linked-Equations) for short, was proposed in reference [9]. Subsequently, it was extended to the 3D staggered/collocated grid systems. IDEAL includes inner doubly-iterative processes for solving pressure equations at each iteration level, and it could adjust the inner iteration times to control the convergence rate and the stability of iteration process, which is greatly different from other segregated algorithms. The objective of this paper is to analyze the effects of inner iteration times on the performance of IDEAL by four incompressible fluid flow problems, two of which belong to open systems, and the others refer to closed systems. It is found that: (1) the robustness of IDEAL is enhanced greatly with the increase of inner iteration times; (2) at the same time step multiple, the outer iteration number decreases with the increase of inner iteration times and the computation time is not related to the inner iteration times; (3) at the optimal time step multiple, the large inner iteration times of 4&4 and 7&7 could reduce the outer iteration number by about 70% and the computation time by about 40% over the small inner iteration times of 1&1.

1. Introduction

In 1972 the first pressure-correction method, SIMPLE, was proposed by Patankar and Spalding [1]. The major approximations of SIMPLE are: (1) the initial pressure field and velocity field are assumed independently; (2) the effect of velocity corrections of the neighboring grids is not considered to simplify the solution procedure. These two approximations do not affect the final solutions when the solution is converged [2], but influence the convergence rate and stability of the solution. Since the birth of SIMPLE, many modified methods, such as SIMPLER [3], SIMPLEC [4], SIMPLEX [5], PISO [6], CLEAR [7,8] etc., have been proposed to overcome the shortcomings of the two approximations. Unlike other algorithms, CLEAR does not introduce the pressure correction, improving the intermediate velocity by solving a pressure equation to make the algorithm fully implicit since there is no term omitted in the derivation process. However, the robustness of CLEAR is somewhat weakened by directly solving the pressure equation. To overcome this disadvantage, IDEAL (Inner Doubly-iterative Efficient Algorithm for Linked-equations) was proposed in [9,10]. Subsequently, it was extended to the 3D staggered/collocated grid systems [11–13].

IDEAL includes inner doubly-iterative processes for solving pressure equation at each iteration level. The first inner iteration time N1 and the second inner iteration time N2 (hereafter named as N1&N2) could be adjusted to control the convergence rate and the stability of iteration process, which is significantly different from other segregated algorithms. In previous articles about IDEAL [11–13], different N1&N2 are applied corresponding to different ranges of time step multiples. As the most crucial adjustable parameters, N1&N2 have great effect on the performance of IDEAL. However, there is little analysis in this aspect. In order to gain further insight into IDEAL, we study the effect of N1&N2 on the performance of the algorithm systematically in this paper.

2. Brief review of IDEAL

The details of the implementations of IDEAL for incompressible steady laminar flow in 3D Cartesian coordinates have been well-documented in reference [11]. Here, we briefly review the solution process of IDEAL as follows.

Step-1: Assume an initial velocity field.
Step-2: Calculate the coefficients and source terms of the discretized momentum equations based on the initial velocity field.
Step-3: Solve the pressure equation iteratively until the iteration time equals to the pre-specified value of N1. Once the first inner

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iteration process for solving pressure equation is over, the final temporary pressure is regarded as the initial pressure.

Step-4: Solve the discretized momentum equations based on the initial velocity and the initial pressure, and obtain the intermediate velocity.

Step-5: Solve the pressure equation iteratively until the iteration time equals to the pre-specified value of N2. Once the second inner iteration process for solving pressure equation is finished, the final temporary velocity is taken as the final velocity of the current iteration level.

Step-6: Regard the final velocity as the initial velocity of the next iteration level, and then return to the step 2 for the next iteration level. Repeat the iterative procedure until convergence is reached.

3. Computation conditions and convergence criteria

The SGSD scheme [14] is adopted for the convective terms in momentum equations. The pressure under-relaxation factor α_P is set as a fixed value of 1. And the same value is used for the u, v, w velocity under-relaxation factors, i.e. $\alpha_u = \alpha_v = \alpha_w$. For convenience of presentation, we use the time step multiple E [4] which relates to the velocity under-relaxation factor $\alpha_{u,v,w}$ by Eq. (1):

$$E = \frac{\alpha_{u,v,w}}{1-\alpha_{u,v,w}} \left(0 < \alpha_{u,v,w} < 1\right) \tag{1}$$

It can be seen that with the time step multiple, we have a much wider range to show the performance of IDEAL in the high-value region of the under-relaxation factor.

In this paper, the convergence criteria require that both the relative maximum residuals of mass (Rs_{Mass}) and momentum (Rs_{UMom}, Rs_{VMom} and Rs_{WMom}) are smaller than the pre-specified values [11].

4. Numerical analysis

In the following part, effects of N1 and N2 on the performance of IDEAL are analyzed by four problems, which are

(1) Problem 1: three-dimensional lid-driven cavity flow;
(2) Problem 2: three-dimensional lid-driven cavity flow with complicated structure;
(3) Problem 3: laminar flow over a backward-facing step;
(4) Problem 4: laminar flow through a duct with complicated structure.

Problems 1 and 2 belong to closed system; problems 3 and 4 refer to open system. The assumptions involved in these problems are laminar, incompressible, steady-state, and constant physical properties of fluid.

4.1. Problem 1: three-dimensional lid-driven cavity flow

This problem belongs to a simple closed system. Fig. 1(a) shows the flow configuration. Calculations are conducted for $Re = 100$ and grid number $= 52 \times 52 \times 52$. All of the residuals Rs_{Mass}, Rs_{UMom}, Rs_{VMom} and Rs_{WMom} are smaller than 10^{-8}. The Reynolds number is defined as

$$Re = \frac{u_{lid} H}{\nu} \tag{2}$$

Table 1 presents v-velocity values along the x-direction center line. The velocity values computed at different N1&N2 are excellently consistent with each other. Therefore, different N1&N2 have no influence on the final solutions if the convergence is reached.

Fig. 1(b) and (c) shows the outer iteration number and the computation time of IDEAL at different N1&N2. Four conclusions can be drawn as follows.

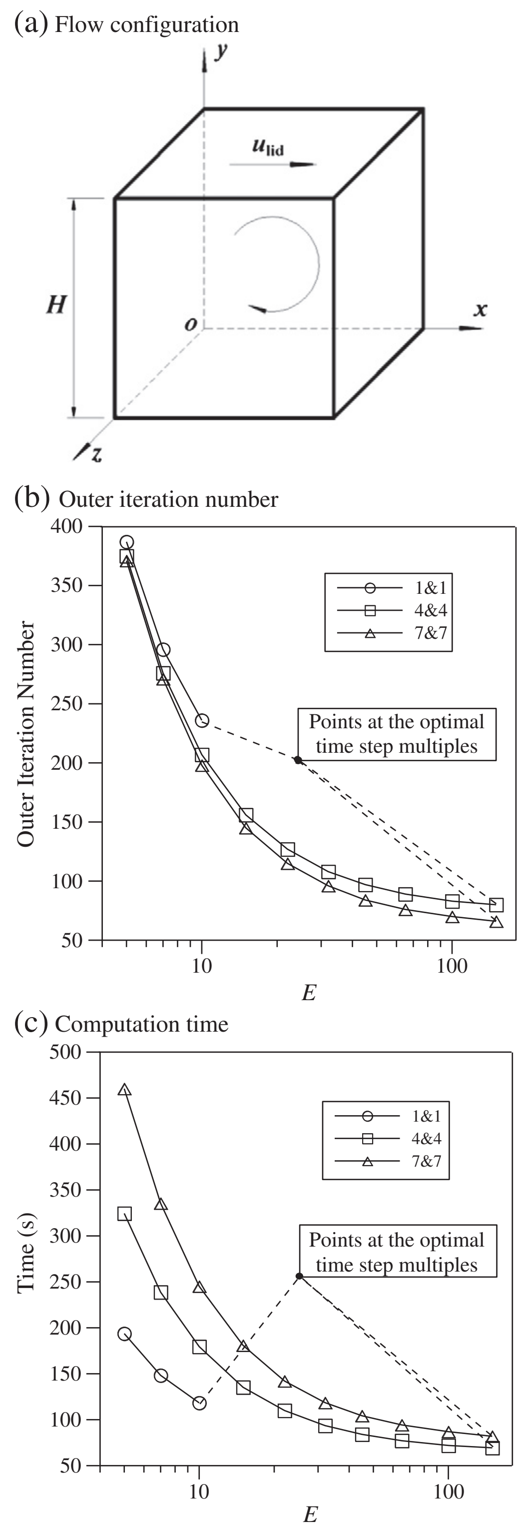

Fig. 1. Flow configuration of 3-D lid-driven cavity flow and the outer iteration number and computation time of IDEAL at different N1&N2 (a) Flow configuration (b) Outer iteration number (c) Computation time.

(1) With the increase of N1&N2, the robustness of IDEAL is enhanced greatly. In the case of 1&1, IDEAL could be converged only at very small time step multiples ($E \leq 10$). In the cases of 4&4 and 7&7, IDEAL can obtain the convergence results at very large time step multiples ($E \leq 150$).

Table 1
Comparison of *v*-velocity values along the *x*-direction center line at different N1&N2.

x coordinates (m)	*v*-velocity values (m/s)		
	1&1	4&4	7&7
0.0	0	0	0
0.1	0.1192719	0.1192733	0.1192697
0.2	0.1514037	0.1514064	0.1514034
0.3	0.1324101	0.1324190	0.1324179
0.4	0.0841729	0.0841875	0.0841879
0.5	0.0132380	0.0132558	0.0132567
0.6	−0.0797810	−0.0797613	−0.0797599
0.7	−0.1837098	−0.1836824	−0.1836803
0.8	−0.2467483	−0.2467422	−0.2467420
0.9	−0.1829373	−0.1829140	−0.1829121
1.0	0	0	0

(2) At the same time step multiple, the outer iteration number decreases with the increase of N1&N2, as shown in Fig. 1(b).
(3) At the same time step multiple, the computation time increases with the increase of N1&N2, as shown in Fig. 1(c).
(4) At the optimal time step multiple, the outer iteration number in the cases of 4&4 and 7&7 decreases by 66.1% and 72.0% over the case of 1&1 respectively, and the computation time is reduced by 41.5% and 30.5% respectively. Here, the optimal time step multiple refers to the point at which the convergence rate is the fastest.

4.2. Problem 2: three-dimensional lid-driven cavity flow with complicated structure

This problem refers to a complicated closed system. Fig. 2(a) shows the flow configuration with three blocks of baffle plates inserted in the cubic cavity. Calculations are conducted for $Re=100$ and grid number$=52\times52\times52$. The Reynolds number is defined in Eq. (2). All of the residuals Rs_{Mass}, Rs_{UMom}, Rs_{VMom} and Rs_{WMom} are smaller than 10^{-8}.

Fig. 2(b) and (c) shows the outer iteration number and the computation time of IDEAL at different N1&N2. It is seen from Fig. 2(b) that the robustness is enhanced and the outer iteration number decreases with the increase of N1&N2. Especially when N1&N2 are changed from 1&1 to 4&4, the robustness is enhanced greatly and the outer iteration number is decreased significantly. At the same time step multiple, the computation time is varied irregularly with the increase of N1&N2, sometimes decreasing and sometimes increasing, as shown in Fig. 2(c). At the optimal time step multiple, the outer iteration number in the cases of 4&4 and 7&7 decreases by 69.6% and 79.9% over the case of 1&1 respectively, and the computation time is reduced by 52.4% and 56.3% respectively.

4.3. Problem 3: laminar flow over a backward-facing step

Configuration shown in Fig. 3(a) belongs to a simple open system, which has been widely used as a typical configuration in computational fluid dynamics study. Calculations are performed for $Re=100$ and grid number$=127\times32\times63$. The inflow velocity distribution is cited from Shah and London [15], and the fully-developed boundary condition is used at the outflow boundary. All of the residuals Rs_{Mass}, Rs_{UMom}, Rs_{VMom} and Rs_{WMom} are set to be less than 10^{-7}. The Reynolds number is defined as

$$Re = \frac{u_{in}H}{\nu} \quad (3)$$

Table 2 presents the reattached length L_R on plane $z=4H$. The results computed at different N1&N2 approach those cited from reference [16], verifying again that different N1&N2 have no influence on the final solutions if the convergence is reached.

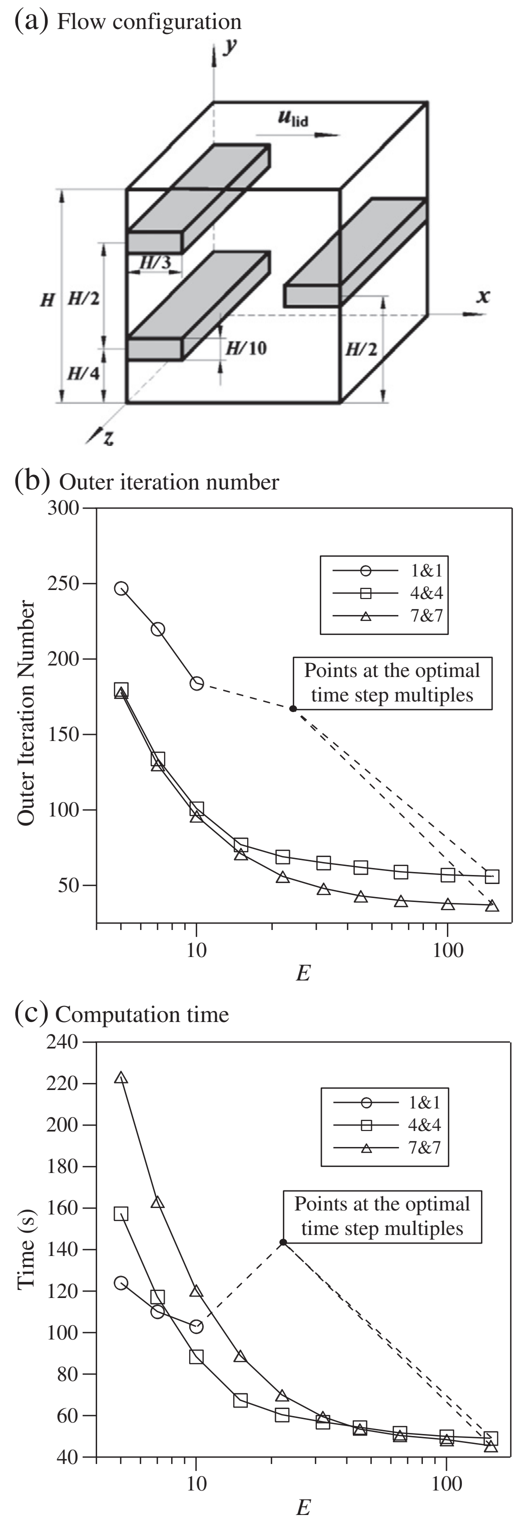

Fig. 2. Flow configuration of 3-D lid-driven cavity flow with complicated structure and the outer iteration number and computation time of IDEAL at different N1&N2 (a) Flow configuration (b) Outer iteration number (c) Computation time.

Fig. 3(b) and (c) shows the outer iteration number and the computation time of IDEAL at different N1&N2. The convergence ranges in the cases of 1&1, 4&4 and 7&7 are $E\leq10$, $E\leq70$, and $E\leq150$ respectively. It proves that the larger the N1&N2 are, the better the robustness of IDEAL performs. At the same time step multiple, the

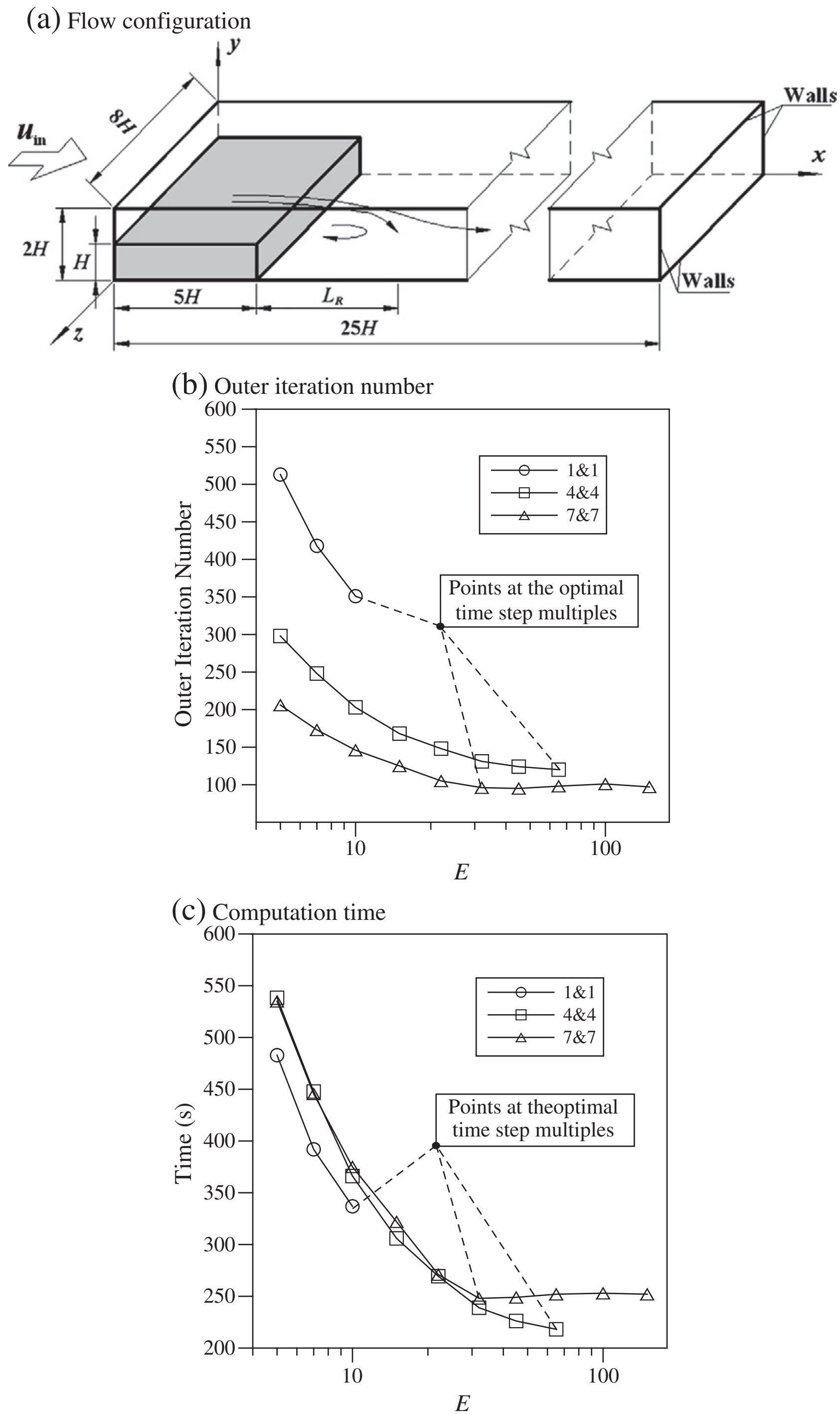

Fig. 3. Flow configuration of laminar fluid flow over a backward-facing step and the outer iteration number and computation time of IDEAL at different N1&N2 (a) Flow configuration (b) Outer iteration number (c) Computation time.

Table 2
The predicted reattached lengthen L_R on plane $z=4H$.

Re	L_R			
	1&1	4&4	7&7	Cited from reference [16]
100	2.895	2.898	2.896	2.854
150	3.998	3.995	3.996	3.972
200	5.051	5.055	5.052	5.052
250	6.059	6.059	6.058	6.055
300	7.010	7.014	7.012	6.981

variation laws of outer iteration number and computation time are identical to those in Problem 1 (simple closed system). At the optimal time step multiple, the outer iteration number in the cases of 4&4 and 7&7 is decreased by 65.8% and 72.9% over the case of 1&1 respectively, and the computation time is reduced by 35.1% and 26.2% respectively.

4.4. Problem 4: laminar flow through a duct with complicated structure

The problem 4 deals with a complicated open system. Fig. 4(a) shows the flow configuration with three blocks of baffle plates inserted in the duct. Calculations are conducted for $Re=100$ and grid

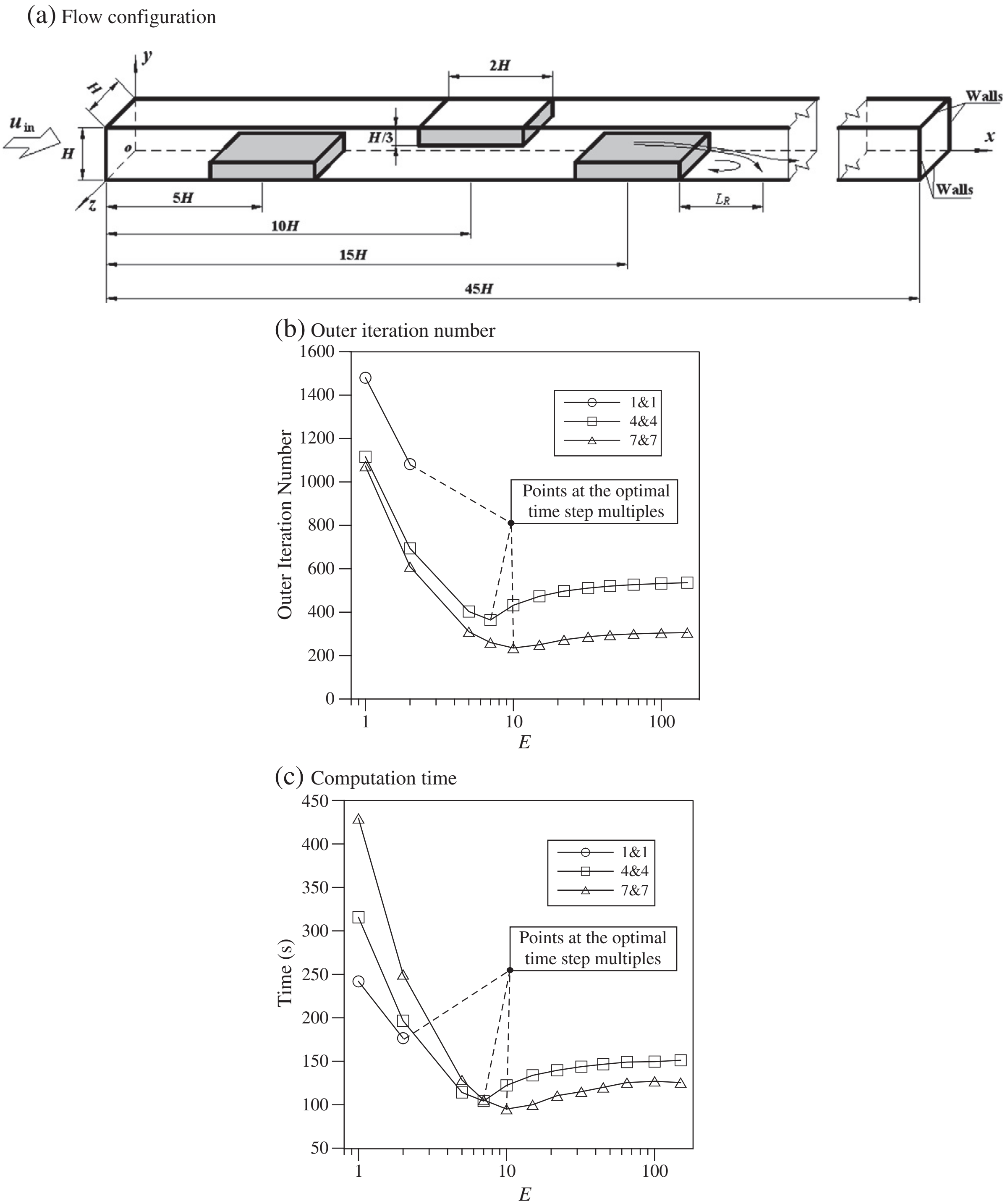

Fig. 4. Flow configuration of laminar fluid flow through a duct with complicated structure and the outer iteration number and computation time of IDEAL at different N1&N2.

number = 150 × 20 × 20. The Reynolds number is defined in Eq. (3). The inflow velocity is uniform, and the fully-developed boundary condition is used at the outflow. All of the residuals Rs_{Mass}, Rs_{UMom}, Rs_{VMom} and Rs_{WMom} are less than 10^{-7}.

Fig. 4(b) and (c) shows the outer iteration number and the computation time of IDEAL at different N1&N2. The variation laws of the robustness, the outer iteration number and the computation time are almost identical to those in Problem 2 (complicated closed system). At the optimal time step multiple, the outer iteration number in the cases of 4&4 and 7&7 decreases by 66.4% and 78.3% over the case of 1&1 respectively, and the computation time is reduced by 40.9% and 46.0% respectively.

5. Conclusion

In this paper, the effect of N1&N2 on the performance of IDEAL is systematically analyzed by four incompressible fluid flow problems. The conclusions are summarized as follows.

(1) The robustness of IDEAL is enhanced greatly with the increase of N1&N2. When N1&N2 equal to 4&4 or 7&7, IDEAL can converge almost at any time step multiple in the four problems above.
(2) At the same time step multiple, the outer iteration number decreases with the increase of N1&N2.

(3) At the same time step multiple, it is not certain about the variation laws of computation time with the increase of N1&N2, sometimes decreasing and sometimes increasing.
(4) At the optimal time step multiple, the large inner iteration times of 4&4 and 7&7 can simultaneously reduce the outer iteration number and the computation time over the small inner iteration times of 1&1. In the four problems above, the outer iteration number and the computation time in the cases of 4&4 and 7&7 can decrease about by 70% and 40% over the case of 1&1 respectively.

The reasons causing the four results above are summarized as three respects:

(1) The larger the N1&N2 are, the better the coupling between velocity and pressure is guaranteed, which explains that the robustness increases and the outer iteration number decreases with the increase of N1&N2.
(2) At the same time step multiple, the outer iteration number decreases, but the computation time spent on each iteration level increases with the increase of N1&N2. Here, the computation time equals to the product of the outer iteration number and the computation time spent on each iteration level, so it is not certain about the variation laws of computation time.
(3) At the optimal time step multiple, larger N1&N2 can simultaneously reduce the outer iteration number and the computation time over 1&1, which is different from the situation of the same time step multiple. This is because the robustness of IDEAL is enhanced greatly with the increase of N1&N2. Thus we can obtain fast convergence result at larger time step multiple. Therefore, a fast convergence rate can be obtained by setting larger values for the inner iteration times and the time step multiple.

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